RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [2017-20]
B.A./B.Sc. FIRST SEMESTER (July – December) 2017
Mid-Semester Examination, September 2017

Date : 15/09/2017

Time : 12 noon – 1 pm

MATH FOR INDUSTRIAL CHEMISTRY (General)

Paper : I

Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

(*Answer **any five** questions*) [5×3]

1. If $a = \cos\alpha + i\sin\alpha$ and n be a positive integer, then prove that

    $a^n + \dfrac{1}{a^n} = 2\cos n\alpha$ and $a^n - \dfrac{1}{a^n} = 2i\sin n\alpha$.

    Hence show that $64\sin^4\alpha\cos^3\alpha = \cos 7\alpha - \cos 5\alpha - 3\cos 3\alpha + 3\cos\alpha$.

2. If the equation $x^4 + ax^3 + bx^2 + cx + d = 0$ has three equal roots, then show that each of them is equal to $\dfrac{6c - ab}{3a^2 - 8b}$.

3. Find the equation whose roots are the squares of the roots of the equation $x^4 - 2x^3 + 3x^2 - x + 7 = 0$.

4. Solve the equation $x^3 - 6x - 9 = 0$ by Cardan's method.

5. If A_i, B_i, C_i be the respective cofactors of a_i, b_i, c_i (i = 1, 2, 3) in $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then show that

    $\begin{vmatrix} B_1 + C_1 & C_1 + A_1 & A_1 + B_1 \\ B_2 + C_2 & C_2 + A_2 & A_2 + B_2 \\ B_3 + C_3 & C_3 + A_3 & A_3 + B_3 \end{vmatrix} = 2\Delta^2$.

6. Investigate for what values of λ and μ, the following equations

    $x + y + z = 6$

    $x + 2y + 3z = 10$

    $x + 2y + \lambda z = \mu$

    have, (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions.

7. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, then use Cayley-Hamilton theorem to express $(A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2)$ as a linear polynomial in A.

Group – B

(*Answer **any two** questions*) [2×5]

8. If the values of a variable are in G.P., then prove that A.M., G.M. and H.M. of the values are also in G.P.

9. The marks obtained by 116 students were grouped in a frequency table in class-intervals of 10 marks each, and the median of this distribution was found to be 35·5. On subsequent verification, it

was found that two observations belonging to the class-interval (31 – 40) were included in the class-interval (21 – 30) by mistake. After correction of this error, the median was found to be 36·5. Calculate the correct frequency in the class-interval (31 – 40).

10. The frequency distribution of a continuous variable, after change of origin, is represented below:

u	:	−30	−20	−10	0	10	20	30
f	:	2	4	8	27	18	15	6

If the mean of the original frequency distribution is 60, find the original frequency distribution.

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